**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

From the given data *μ* = 45

*σ* = 8

Given that 1 hour from drop-off that means 60 min and work will begin 10 min after the car drop-off

And X = 50

Z – score would be (X - µ) / σ => (50-45)/8 => 0.625

In Python code we can use cumulative density function to find out the remaining part in the distribution of the data.

P(X>50) = 1 - P(X<=50)

1 – 0.734 => **0.266**

So the answer is option B => **0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

From the given data we have *μ* = 38

*σ* = 6

Here also by using python code we can use Cumulative density function

Ans A) Employee older than 44 years of age then

1 – stats.norm.cdf(44, loc=38, scale=8)

**0.1586 =>** This is X > 44

P (38 < X < 44) would be stats.norm.cdf(44,38,6) - stats.norm.cdf(38,38,6)

**0.341**

Ans B) Under the age of 30 then P( X< 30)

stats.norm.cdf(30, 38,6)

**0.0912**

And the number of employees attending training program from 400 is N\*P(X<30)

400 \* stats.norm.cdf(30, 38, 6)

**36.4844**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

If X1 and X2 are independent and identically distributed (i.i.d.) normal random variables with the same mean (μ) and the same variance (σ^2), then let's examine the difference between 2X1 and X1 + X2 in terms of their distributions and parameters.

1. Distribution of 2X1:

2X1 is a scaled version of X1, where each value of X1 is doubled.

The mean of 2X1 is 2μ because you're simply scaling the mean by a factor of 2.

The variance of 2X1 is 4σ^2 because when you scale a random variable by a constant (in this case, 2), the variance is scaled by the square of that constant.

So, for 2X1:

Mean (μ\_2X1) = 2μ

Variance (σ^2\_2X1) = 4σ^2

1. Distribution of X1 + X2:

X1 + X2 is the sum of two independent normal random variables.

The mean of the sum is the sum of the individual means, which is 2μ because μ + μ = 2μ.

The variance of the sum is the sum of the individual variances, which is 2σ^2 because σ^2 + σ^2 = 2σ^2.

So, for X1 + X2:

Mean (μ\_X1+X2) = 2μ

Variance (σ^2\_X1+X2) = 2σ^2

In summary, when you take the difference between 2X1 and X1 + X2:

The distribution of 2X1 will have a mean that is twice the mean of X1 and a variance that is four times the variance of X1.

The distribution of X1 + X2 will have a mean that is twice the mean of X1 and a variance that is two times the variance of X1.

In both cases, the means are the same (2μ), but the variances are different. The variance of 2X1 is larger (4σ^2) compared to the variance of X1 + X2 (2σ^2). This means that 2X1 will generally have larger fluctuations around the mean compared to X1 + X2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

By using python code we can do this problem

We can use interval from stats to find out the solution

np.round(stats.norm.interval(0.99, loc=100, scale = 20), 3)

And the answer is **48.483, 151.517**

That mean **Option D** is the correct answer.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

we need to first calculate the total profit of the company in Rupees, convert the given profit distributions from dollars to rupees, and then perform the necessary calculations.

Given:

Profit1 ~ N(5, 32) in $ Million

Profit2 ~ N(7, 42) in $ Million

Conversion rate: $1 = Rs. 45

Let's calculate the total profit in Rupees:

A. To specify a Rupee range centered on the mean with 95% probability, we can use the z-score approach for a normal distribution.

For Profit1:

Mean1 = 5 million dollars

Variance1 = 32 million dollars^2

Standard Deviation1 = sqrt(32) million dollars

For Profit2:

Mean2 = 7 million dollars

Variance2 = 42 million dollars^2

Standard Deviation2 = sqrt(42) million dollars

Now, let's calculate the total profit in dollars:

Total Profit in $ = Profit1 + Profit2

Total Profit in $ = (5 + 7) million dollars = 12 million dollars

Now, convert the total profit to Rupees using the conversion rate:

Total Profit in Rupees = Total Profit in $ \* Rs. 45

Total Profit in Rupees = 12 million dollars \* Rs. 45 = Rs. 540 million

Now, we'll calculate the standard deviation of the total profit in Rupees:

Standard Deviation in Rupees = Standard Deviation in $ \* Rs. 45

Standard Deviation in Rupees = sqrt(32) million dollars \* Rs. 45 = Rs. 180 sqrt(2) million ≈ Rs. 254.55 million

To find the Rupee range centered on the mean that contains 95% probability, we'll use the z-score for a 95% confidence interval (which corresponds to approximately ±1.96 standard deviations for a normal distribution):

Lower Limit = Mean - 1.96 \* Standard Deviation

Upper Limit = Mean + 1.96 \* Standard Deviation

Lower Limit = Rs. 540 - 1.96 \* Rs. 254.55 ≈ Rs. 35.23 million

Upper Limit = Rs. 540 + 1.96 \* Rs. 254.55 ≈ Rs. 1044.77 million

So, the Rupee range centered on the mean that contains 95% probability for the annual profit of the company is approximately Rs. 35.23 million to Rs. 1044.77 million.

B. To specify the 5th percentile of profit in Rupees for the company, we'll find the z-score corresponding to the 5th percentile (which is -1.645 for a one-tailed 5% confidence interval):

5th Percentile = Mean + (Z \* Standard Deviation)

5th Percentile = Rs. 540 + (-1.645 \* Rs. 254.55) ≈ Rs. 96.04 million

So, the 5th percentile of profit for the company is approximately Rs. 96.04 million.

C. To determine which of the two divisions has a larger probability of making a loss in a given year, we need to calculate the probability of each division having a negative profit in Rupees.

For Profit1:

Mean1 = 5 million dollars = Rs. 225 million

Standard Deviation1 = Rs. 180 sqrt(2) million ≈ Rs. 254.55 million

Now, calculate the z-score for a loss (profit < 0):

Z1 = (0 - Rs. 225 million) / Rs. 254.55 million ≈ -0.882

Using the standard normal distribution table or calculator, find the probability that Z1 is less than -0.882. This represents the probability of Profit1 being negative.

For Profit2:

Mean2 = 7 million dollars = Rs. 315 million

Standard Deviation2 = Rs. 180 sqrt(2) million ≈ Rs. 254.55 million

Calculate the z-score for a loss (profit < 0):

Z2 = (0 - Rs. 315 million) / Rs. 254.55 million ≈ -1.238

Using the standard normal distribution table or calculator, find the probability that Z2 is less than -1.238. This represents the probability of Profit2 being negative.

Compare the probabilities obtained for Profit1 and Profit2. The division with the higher probability of making a loss in a given year has a larger likelihood of making a loss.